Groundwater Flow in Fractured Rock Matrix: Application ofthe Forchheimer Model

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Abstract

Groundwater flow through pores and fractures is driven by both pressure and hydraulic head. Upon this, different models have been developed. In the models, some parameters influencing the pressure and hydraulic head are embedded. An examination of the flow behaviour of water in pores and fractures must take cognizance of their roles. This paper studies the groundwater flow through a fractured aquifer using the Forchheimer model, to check the sustainability of any water project that may depend on the groundwater flow. The problem is solved using the Separation of Variables and Perturbation approaches. Solutions of the hydraulic head are obtained and presented quantitatively and graphically. The analysis of results shows that the increase in themagnitude of the specific discharge does not affect the hydraulic head in the flow through the fractured unconfined aquifers, but causes fluctuation in the hydraulic headin the flow through the fracture running from the confined to the unconfined aquifer. Furthermore, the increase in the thickness of the sub-layers of the aquifer increases the hydraulic head in the fractured unconfined aquifers, but causes fluctuation in the hydraulic head structure in the flow through the fracture running from the confined to the unconfined aquifer.

Keywords: *Forchheimer Model, Fractured aquifer, Groundwater modelling, Hydrogeology, Managed/artificial reservoir*

1 Introduction

Modelling of groundwater flow has applications in drainage, dam stability, management of landslides, etc.

There are three facets of groundwater: the connate water, which is associated with petroleum reservoirs; the juvenile/magmatic water, which is associated with the Earth crust and are released into the atmosphere during volcanic eruptions, and that in the water saturated subsurface zones called the aquifers, and which is formed from the meteoric and snow melts that penetrate the soil, cracks and faults. Importantly, the aquifer groundwater is a source of freshwater.

As geologic formations, aquifers are liable to fracturing. They can fracture naturally due to the movement of the earth (plate tectonics) or artificially through the application of technology. An aquiferfractures when a stress higher than the compositional rock strength acts on it, and causes it to lose its cohesive force along its weakest plane (Park 2005). At fracturing, channels are created in between the parts. The fracture may occur in the confined, unconfined, or both, and in which case the fracture runs from the confined aquifer through the unconfined aquifer. The groundwater flow in the fractured aquifers is laminar but faster than those in pores. Upon this, the groundwater flow in the aquifers can be considered in two perspectives: the flow through pores of porous media, and that through fractured rock matrixes.

In another development, models (mathematical and simulative) are used to study and predict the state (effectiveness or malfunctioning) of any system. They are developed based on the existing situations, and new ones are built on the older ones. Based on this, models have been developed for studying groundwater flow (through pores and channels) in the aquifers.

Some research reports exist on groundwater flow through fractures/channels. For instance,Shi et al (2018), using the Forchheimer model, studied numerically the groundwater flow through a fault.Mwetulundila and Atangana (2020) studied the groundwater flow through fractured artificially recharged aquifers using the Forchheimer model and numerical solution approaches. They used it to study and manage the water-stressedsituations in the Windhoek,the Namibian capital city. Windhoek only source of water supply is dams, which are faced with serious evaporation problem. Upon these, multiple water augumentation techniques were proposed and implemented.El-Kharakany et al (2022) studied modeling radial groundwater flow in fractured media using fracture continuum approach; Zhou et al (2023) studied groundwater flow through fractured rocks and seepage control in geotechnical engineering.

FromMwetulundila and Atangana (2020),fractured empty equifers, or fractured aquifers with limited quantity of water exist. This work studies the sustenability ofwater supply from a naturally recharged fractured unconfined and confined aquifers to an artificial surface reservoir using the Forchheimer model and analytic solution approaches.

2. Physics of Problem and Mathematical Formulation

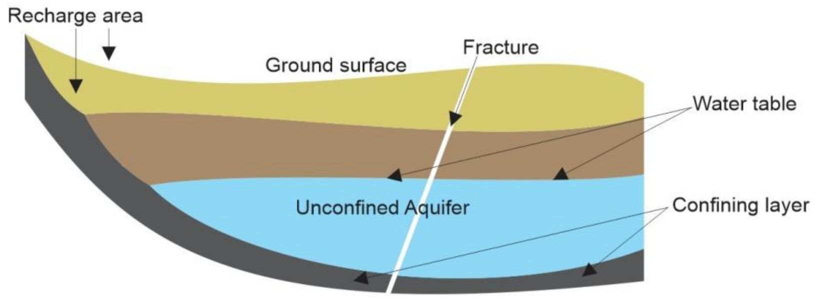

Fig. 1 A Schematic of a Fracture through the Unconfined Aquifer

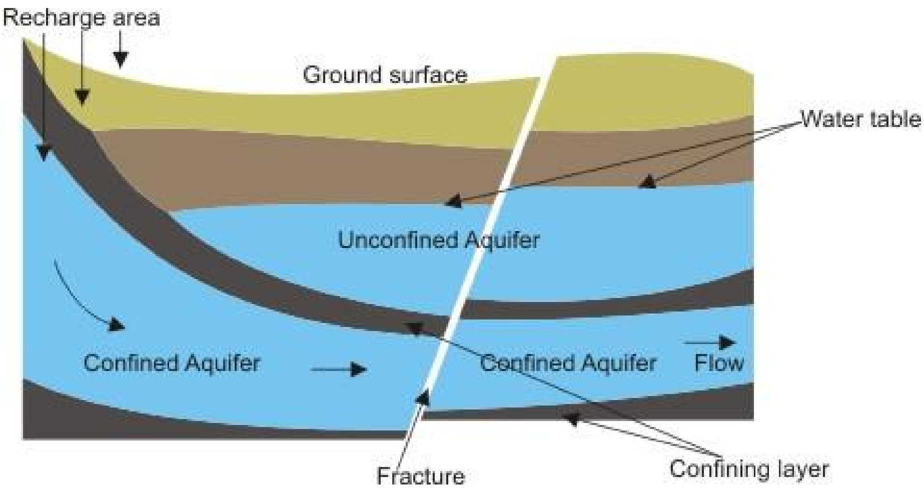

Fig. 2 A Schematic of a Fracture through the Unconfined-Confined Aquifers

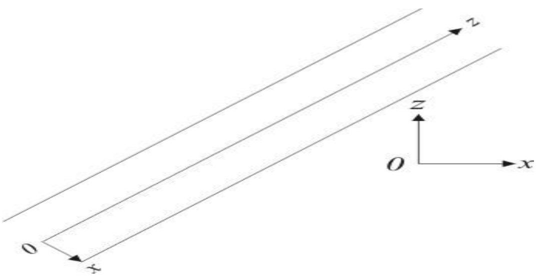

Fig. 3 A Schematic Mathematical Representation of the Flow through the Fracture

A one-dimensional time-dependent mathematical model of groundwater flow in fractured aquifers using the Forchheimer assumptions is considered. The schematics of the flow are shown in Fig. 1 - Fig. 3. This model is developed based on the assumptions that:(a) the unconfined aquifer can fracture independent of the confined aquifer (b) the water in the fractured confined aquifer cannot be obtained without going through the unconfined aquifer, hence, the fracture runs from the confined to unconfined; the fracture is rectangular and symmetrical, or likened to a two vertical parallel plates channel of finite width, but infinite length; the water in fracture flows into a managed/artificial sink/reservoir some distance away from the source;the flow of water in the aquifer is predominantly horizontal; the effect of the slight angle of elevation of the channel on the flow is negligible, thus back-flow is prevented; a perturbation occurs at the point where the water from both aquifers meet. Using the hydraulic head-dependent model, if (x,t) is the spatial axial coordinate, and h_f is the hydraulic head of the fracture, then the governing equations are:

For the unconfined aquifer,

$$S_m \frac{\partial h_m}{\partial t} = DK_m \frac{\partial^2 h_m}{\partial x^2} + V_r \quad (1)$$

where

$$V_r = \xi T_m \left(h_{fi} - h_{mi}\right)$$

is the fluid exchange rate between the pores and the fractures, D is the depth of the elementary volume/aquifer thickness, K_m is the hydraulic conductivity, S_m is the storage coefficient in aquifer; f represents the fracture, m represents the rock matrix, $T_m = DK_m$ is the transmissivity of the aquifer (a measure of the flow rate of groundwater per unit width of an aquifer per unit hydraulic head gradient. It is used to predict groundwater movement, determine the safe yield of an aquifer, and to calculate the yield of a borehole), ξ is the fluid transfer parameter (see Mwetulundila and Atangana, 2020).

and for the confined aquifer

$$S_m \frac{\partial h_m}{\partial t} = DK_m \frac{\partial^2 h_m}{\partial x^2} \quad (2)$$

The difference between equations(1) and (2) is that the fluid transfer/exchange rate V_r between the impermeable rock matrix and the fracture is zeroin equation (2).

The analysis of the groundwater flow in fractured confined aquifers is based upon the assumptions that: there is no groundwater within the rock matrix, therefore, there is no significant storage there.

Boundary Conditions

Due to fluid-particle interaction, pressures are exerted on the walls. The hydraulic head at the walls of the unconfined and confined aquifers is normal, and the boundary conditions are:

$$h_f(0,t) = 1 \quad (3)$$

$$h_f(L,t) = h_{w1}, h_{w1} > 1, \quad (4)$$

for the unconfined aquifer, and

$$h_f(0,t) = \omega \quad (5)$$

$$h_f(L,t) = h_{w2}, h_{w2} < \omega, \quad (6)$$

for the confined aquifer.

Forchheimer Model for Groundwater Flow through Fractured Aquifers

Forchheimer prescribed the term

$$J = aq + bq^2 = (a + bq)q = F(q)q \quad (7)$$

where q is the magnitude of the specific discharge in the coordinate direction, such that in a one-directional flow in the x-direction, say, $q = q_x$; $F(q)$ is a scalar function of the q at any point; $F(q) = (a + bq)$ is used for the high velocity of water within the fracture; a and b are the Forchheimer linear and non-linear parameters, a depends on the fluid properties, and b the media properties like porosity. The quadratic term (q^2) of the Forchheimer equations is related to the inertial effect in the laminar regime.

Substituting equation (7)into equations (1) and (2), respectively, we obtain

$$\frac{\partial h_f}{\partial t} = \alpha_1 \frac{\partial^2 h_f}{\partial x^2} + \beta \quad (8)$$

for the flow in the unconfined fractured aquifers,

where $\alpha_1 = \frac{D}{(a+b)S_f}, \beta = \frac{V_r}{S_f}$

and

$$S_f \frac{\partial h_f}{\partial t} = \eta \frac{\partial^2 h_f}{\partial x^2} \tag{9}$$

for the confined fractured aquifers,

where $\eta = \frac{D}{bqS_f}$

.

The difference between equation (8) and (9) is that in equation (9) the linear parameter a of the Forchheimer equation and V_r are zero.

3. Method of solution

Equations (8) and (9) are solved subject to equations (3) - (6)analytically using the method of Separation of Variables.

$$h_{fu} = a_1 e^{-\lambda^2 t}\left(-A_1 e^{-x\xi} + B_1 - \frac{x\gamma}{a_1 e^{-\lambda^2 t}}\right) = e^{-\lambda^2 t}\left(-Ae^{-x\xi} + B - \frac{x\gamma}{e^{-\lambda^2 t}}\right) \tag{10}$$

where

$$A = \frac{\xi}{e^{-\lambda^2 t}\left(e^{-L\xi} - 1\right)}\left(1 - h_{w1} - \frac{L\gamma}{\xi}\right)$$

$$B = \frac{1}{e^{-2\lambda^2 t}} + \frac{\xi}{e^{-\lambda^2 t}\left(e^{-L\xi} - 1\right)}\left(1 - h_{w1} - \frac{L\gamma}{\xi}\right)$$

$$h_{fc} = b_1 e^{-\mu^2 t}\left(C_1 e^{-x\tau} + D_1\right) = e^{-\mu^2 t}\left(Ce^{-x\tau} + D\right) \tag{11}$$

where

$$C = \frac{\omega}{e^{-\mu^2 t}} - \frac{1}{e^{-\mu^2 t}\left(1 - e^{-L\tau}\right)}\left(h_{w2} - \omega e^{-L\tau}\right)$$

$$D = \frac{1}{\left(1 - e^{-L\tau}\right)e^{-\mu^2 t}}\left(h_{w2} - \omega e^{-L\tau}\right)$$

Based on the assumption that the groundwater flows from the confined aquifer through the unconfined aquifer to a distant sink/ reservoir, the point where the water of both aquifersmeet disturbances are generated. Hence, for the combined flow of the water from both aquifers, the Perturbation Method of Solution becomes very suitable for this problem, and is presented as

$$h_f = h_{fc} + \varepsilon h_{fu} \tag{12}$$

where $\varepsilon < 1$, is the perturbation parameter.

4 **Results**

The enhancement of the driving forces that control the groundwater flow accounts for the sustenability of groundwater supply.The effects of the parameters on which the driving forces depend are investigated. For constant values of $\lambda^2 = 1, \mu^2 = 1, a = (\rho) = 0.9, b = (\chi) = 5,$ $\omega = 10, h_{w1} = 0.5, h_{w2} = 5$, $h_{fi} = 10, h_{mi} = 3,$ and varied values of $S_f = 0.01, 0.03, ,0.1, 0.2, 0.3$, $q = 15,30,45,60,75$, $D(=\phi) = 10,20,30,40,50$, $K_m = 0.01, 0.03, 0.1, 0.2, 0.3$, Table 1 and Table 2, and Fig. 4 and Fig. 5are obtained.

Table 1 Hydraulic Head-Discharge Relationship in the Unconfined Aquifer Flow

	hf(q=15)	hf(q=30)	hf(q=45)	hf(q=60)	hf(q=75)
Hf	0.	0.	0.	0.	0.
15	14.0666	14.0666	14.0666	14.0666	14.0666
30	28.1293	28.1293	28.1293	28.1293	28.1293
45	42.1883	42.1883	42.1883	42.1883	42.1883
60	56.2439	56.2439	56.2439	56.2439	56.2439
75	70.2963	70.2963	70.2963	70.2963	70.2963

Table 2 Hydraulic Head-Discharge Relationship in the Combined Unconfined-Confined Aquifer Flow

	hf(q=15)	hf(q=30)	hf(q=45)	hf(q=60)	hf(q=75)
Hf	5.0000000	5.000000	5.000000	5.000000	5.000000
15	2.7714411	2.785717	2.790477	2.792857	2.794286
20	0.5753709	0.587557	0.591676	0.593746	0.594992
30	1.5894817	1.594798	1.596545	1.597414	1.597934
40	3.7243339	3.761660	3.774326	3.780702	3.784541
50	5.8303487	5.913336	5.941805	5.956196	5.964880

The effect of specific discharge on the hydraulic head in the groundwater flow through a fractured aquifer is shown in Table 1 and Table 2. Table 1 shows that an increase in the specific discharge has no effects on the hydraulic heads in the flow through a fractured unconfined aquifer. The hydraulic heads remain constant despite the increasing specific discharge values, and by implication, the sustainability of such a water supply plan cannot be accounted for. Table 2 depicts that the hydraulic head increases as the specific discharge increases in the groundwater flow through a fracture running from the confined via the unconfined aquifer to an artificial surface reservoir. Groundwater discharge is the flow of water from the aquifer into artificial reservoirs in the presence of a hydraulic gradient. This discharge of water from the aquifer is influenced by permeability/porosity, recharge, and hydraulic head. In particular, the relationship between the discharge and hydraulic head is reciprocating. The discharge increases the hydraulic head, and vice versa. Under a favourable recharging climate, as water is discharged from the aquifer, it is recharged to strike a discharge-recharge equilibrium. By implication, with the discharge-recharge ratio at equilibrium, continuous flow is maintained, and thus a continuous water supply is sustained.

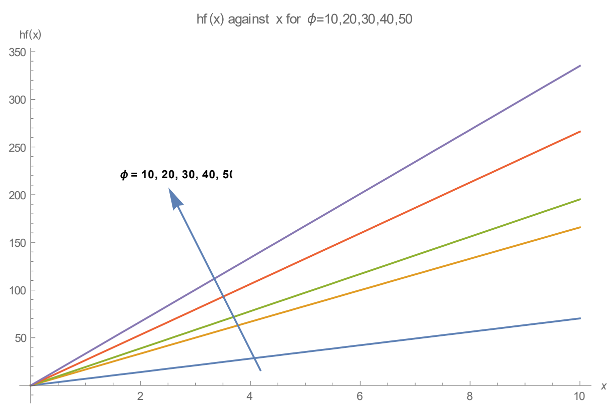

Fig. 4 Hydraulic Head-Thickness of the Saturated Layers Relationship in the Unconfined Aquifer Flow

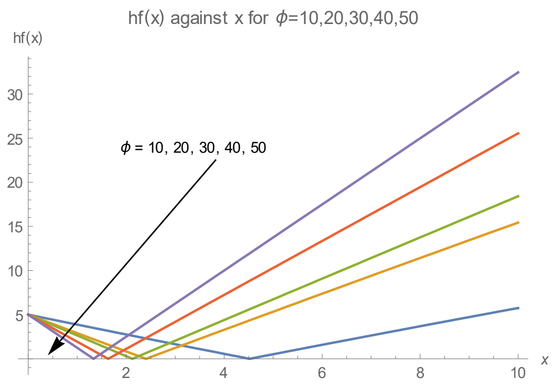

Fig. 5 Hydraulic Head-Thickness of Sublayers Profiles in the Unconfined-Confined Aquifer Flow

More so, the effect of the thickness of the sub-layers of the aquifer on the hydraulic head in the groundwater flow through the fractured aquifers is shown in Fig. 4 and Fig. 5. Fig. 4 shows that an increase in the thickness of the sub-layers of the fractured unconfined aquifer increases the hydraulic head. As the value of the thickness of the sublayers increase the hydraulic head rises. The thickness of the sub-layer of an aquifer determines the volume of water it can hold, and this in turn determines the recharge level/the height of the water column. Therefore, this is very favourable to the sustenability of any water supply plan.

Fig. 5 shows that an increase in the thickness of the sub-layers of the aquifer causes fluctuation in the hydraulic head structure in the groundwater flow through a fracture running from the confined aquifer to the unconfined aquifer. As this thickness increases, the hydraulic head profiles decreasingly drop at different points in the region $1 \leq x \leq 5$, then rise from such points. The fluctuation in the hydraulic head structures leads to lose of energy for the flow, which negatively affects the quantity of water moving into the artificial reservoirs, and which in turn reduces water supply.

The fluctuation in the hydraulic head profiles may be caused by some other factors like the high pressure arising from the confined aquifer.

5. Conclusion

The groundwater flow through fractured aquifers using the Forchheimer model and analytic approaches is investigated. The analysis of results shows that an increase in the:

- Magnitude of the specific discharge does not affect the hydraulic head in the flow through the fractured unconfined aquifers, but causes fluctuation in the hydraulic head in the flow through the fracture running from the confined to the unconfined aquifer.
- Thickness of the sub-layers of the aquifer increases the hydraulic head in the fractured unconfined aquifers, but causes fluctuation in the hydraulic head structure in the flow through the fracuture running from the confined to the unconfined aquifer.

The fluctuation in the driving force leads to loss of enegy for the flow, which has negative effects on flow, and which in turn reduces the sustenance of any water supply plan.

Recommendations

The water sustainability consideration of any water supply plan should be factor/parameter-based. Therefore, all the factors that enhance the hydraulic head need not be present in an aquifer. Based on the analysis of the results, we recommend as follows, considering:

1. The discharge factor, the fracture should be made to run from the confined to the unconfined aquifers.
2. The layer thickness factor, only the unconfined aquifer can be fractured and used, as it gives the best-needed result.

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